AMERICAN UNIVERSITY OF BEIRUT

Suliman S. Olayan School of Business

DCSN 205 – Managerial Decision Making
Fall 2010–2011 Midterm Exam
25 November 2010

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"I fully understand and strongly adhere to the School of Business Honor Code. I vow to complete the exam on my own without giving or receiving help from anyone, and to adhere to the academic integrity standards reflected in the AUB student code of conduct."

Your Signature: ______________________________

Your Name and ID: ______________________________

Select Your Section [1 mark]:

Instructor	Section	Days and Times	Selection
Dr. Neil Yorke-Smith	2	MWF 10:00am–10:50am	☐
	3	MWF 1:00pm–1:50pm	☐
Dr. Walid Nasr	4	TR 9:30am–10:45am	☐
	5	TR 11:00am–12:15pm	☐
Dr. Krzysztof Fleszar	6	TR 2:00pm–3:15pm	☐
	7	TR 3:30pm–4:45pm	☐

- This exam has 5 exercises, for a total of 100 points. You have 2 hours to complete it.
- Write your answers in the spaces provided. Be *concise* and follow the instructions closely. If you run out of room for an answer, continue on the back of the page.
- Your understanding of the questions is part of the exam. No questions will be answered by instructors. If in doubt, write your assumptions and continue solving.
- Do not start the exam (do not turn to the next page) until instructed to do so.
- Once you begin, it is your responsibility to check that your paper contains 13 pages.

Exercise 1

1. What is wrong with the following linear programming model? [2]

$$
\begin{aligned}
\min\ & 20X_{AD} + 15X_{AE} - 5X_{BA} \\
\text{s.t.}\ & X_{BA} - X_{AD} - X_{AE} \leq 25 \\
& -X_{BA} - X_{BE} \leq -40 \\
& -X_{CD} \leq -30 \\
& X_{AD} \times X_{AE} + X_{CD} - X_{DE} \leq 50 \\
& X_{BA} \leq 5 \\
& \text{All } X_{ij} \geq 0
\end{aligned}
$$

2. Which of (1) linear programming, (2) network flow, (3) integer linear programming, and (4) multi-objective linear programming models are the most difficult to *solve* (with the aid of a computer)? Justify your answer. [4]

3. Give and briefly explain one reason why shadow prices are useful in business decision making. [4]

4. Sketch the feasible region of the following model. (You can use a pencil.) [12]

$$
\begin{aligned}
\min\ & 2X_1 - 4.5X_2 \\
\text{s.t.}\ & X_1 - X_2 \leq 3 \\
& 2X_1 + X_2 \geq -1 \\
& X_1 \geq -1 \\
& X_2 \leq 5 \\
& X_1, X_2 \text{ are integer}
\end{aligned}
$$

Exercise 2

A mining company extracts minerals from ore mined at two different sites. Each ton of ore type 1 contains 20% copper, 20% zinc and 15% magnesium. Each ton of ore type 2 contains 30% copper, 25% zinc and 10% magnesium. Ore type 1 costs $90 per ton and ore type 2 costs $120 per ton. We would like to buy enough ore to extract in total at least 8 tons of copper, 6 tons of zinc, and 5 tons of magnesium in the least costly manner.

The following LP model was defined:

X_1 = tons of type 1 ore to be bought
X_2 = tons of type 2 ore to be bought

$$\begin{aligned} \min\ & 90X_1 + 120X_2 \text{ (total cost)} \\ \text{s.t. }\ & 0.2X_1 + 0.3X_2 \geq 8 \text{ (copper constraint)} \\ & 0.2X_1 + 0.25X_2 \geq 6 \text{ (zinc constraint)} \\ & 0.15X_1 + 0.1X_2 \geq 5 \text{ (magnesium constraint)} \\ & X_1, X_2 \geq 0 \end{aligned}$$

The LP model was solved using Excel Solver. The following Sensitivity Report was obtained:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C3	X1	28	0	90	90	10
D3	X2	8	0	120	15	60

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E9	Copper Constraint	8	360	8	7	1.333333333
E10	Zinc Constraint	7.6	0	6	1.6	1E+30
E11	Magnesium Constraint	5	120	5	1	2.333333333

1. What is the optimal solution and total cost? [2]

2. Which constraints are binding and which are non-binding? Calculate the slack for the non-binding constraint(s). [4]

3. Ore type 1 now costs \$110. Is it possible to find the optimal decision and total cost without resolving the LP? If so, find the optimal decision and cost. Explain. [4]

4. Consider the original model (i.e., where ore type 1 costs \$90). The copper requirements have changed and now we need to extract 12 tons instead of 8 tons. Is it possible to find the optimal decision and total cost without resolving the LP? If so, find the optimal decision and cost. Explain. [4]

5. Consider the original model. We are considering buying an additional ore type 3, which contains 30% copper, 20% zinc and 15% magnesium and costs $130 per ton.

(a) Formulate a new LP that takes ore type 3 into consideration. 6

(b) Would you recommend considering to buy any amount of ore type 3? Explain. 4

Exercise 3

A manager of a job shop has to assign jobs to machines. Each job has to be assigned to only one machine. Each machine can process at most four jobs. The time in hours of processing each job depends on the machine on which it is performed and is shown in the table below. The manager would like to assign the jobs such that the sum of processing times is minimized.

	Machines		
Jobs	A	B	C
1	10	14	15
2	12	13	15
3	9	12	12
4	14	16	18
5	7	12	11

1. Draw a network model for the problem. Indicate supplies and demands. Show all costs related to jobs 1 and 2. 6

2. Write a mathematical model for the problem, assuming that the cost of performing job i on machine j given in the table on the previous page is denoted by c_{ij}. 8

3. The manager discovered that there is an additional amount of time that should be taken into account. If a machine is used, then a 2-hour setup has to be performed for it. Note that for each machine, the setup is performed only once, or not at all. If there are jobs assigned to a machine, the setup time is required, but the setup is not performed if no jobs are assigned to that machine. Write all necessary modifications of your model from Part 2. 4

4. Consider again the problem without setups, i.e. as in Part 2. When the problem was solved, four jobs were assigned to machine A and one to machine B. The manager would like to find a more balanced assignment by minimizing the maximum load over all machines (where load of a machine is the total processing time of jobs assigned to this machine). Write all necessary modifications of your model from Part 2. 4

Exercise 4

A dietitian wants to formulate a high calorie sports food product for a customer. The following information is available about the 2 ingredients which can be combined to make the product. The customer wants 1000 kgs of the product and it must contain at least 250 kgs of ingredient 1 and 300 kgs of ingredient 2. The product should contain at most 20% of fat and at least 35% of protein.

	Ingredient 1	Ingredient 2
% Fat	10	25
% Protein	30	40
Calories per kg	3000	2000
Cost per kg	$1.50	$1.00

1. Formulate a linear model that would maximize the total calories in the final product. 7

2. The dietitian is now concerned with minimizing the total cost as well as maximizing the total calorie intake. Write down the two objective functions. [2]

3. When the LP model formulated in Part 1 was solved, the solution with total calories of 2,500,000 and total cost of \$1,250 was obtained. The customer is not satisfied with the total cost and would like to decrease it to \$1,100. Using 2,500,000 calories and \$1,100 as the target values, write the necessary modifications of the model from Part 1 that would allow the customer to solve the multi-objective optimization problem with weights w_1 and w_2 provided by the customer. [6]

Exercise 5

The airline Quantas is configuring a new Airbus 380 plane and must decide how many First-Class seats, Business-Class seats, and Economy-Class seats to allocate. The plane has space for 260 Economy-Class seats. Each First-Class seat takes up the space of 2.5 Economy-Class seats; each Business-Class seat takes up the space of 1.75 Economy-Class seats. The profit on a First-Class ticket is 4 times the profit of an Economy-Class ticket; the profit of a Business-Class ticket is 3 times the profit of an Economy-Class ticket. The number of Economy-Class seats must be at least two times larger than the number of all other seats. Once the number of seats for each class is decided, it cannot be changed.

1. Assuming that Quantas can sell enough tickets to occupy all seats, formulate a model for this problem that will maximize the total profit. 6

2. Quantas is planning to use the plane for one connection. Every day the plane will fly from Melbourne to Kuala Lumpur and back. The expected demand for each class in each direction—on each working day and on each day of the weekend—is provided below. 10

Class	First	Business	Economy
Demand during each working day	25	80	200
Demand during Saturday and Sunday	10	35	250

Assuming that Quantas will not overbook the plane (i.e., it will not sell more tickets than the number of seats for each class), formulate a refined model for this problem that will maximize the total profit per week.